There is also a relation between α and the residual water content or the displacement coefficient. The corresponding equations are

$$\ln \omega = 0.141\alpha + 1.251, \quad \beta = -2.069\alpha + 84.530,$$

where R = 0.74 and R = −0.71, respectively. The residual water content increases with the proportion of stagnant zones (Fig. 3a), while the displacement coefficient decreases (Fig. 3b).

NOTATION

t, time; h, tracer concentration; L, length of porous medium; q, volume flow rate in channels; α, stagnant zones; u, linear flow velocity in channels; P, pressure; i, imaginary unit; ω, residual water content; β, displacement coefficient.

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SIMULATION OF MASS-EXCHANGE PROCESSES IN A MASS-DIFFUSION SEPARATING ELEMENT

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We obtain a system of nonlinear algebraic equations which can be used for calculating the working and separating characteristics of a mass-diffusion element.

The mass-diffusion method has been used for a relatively long time in the production of inert-gas isotopes, but the theory of a separating element has not been adequately worked out up to now, so that it is impossible to design the apparatus in an optimum manner.

In [1] we showed that the fundamental separating characteristics of a mass-diffusion element, ε and L', are determined by its internal working parameters ln q and θ_V. These two parameters were considered to be independent in [1], which enabled us to analyze their effect on ε and L' over wide ranges; however, in a real apparatus a connection between them exists and must be taken into account.

Taking account of such a connection is possible only on the basis of a consideration of a model which will describe the diffusion and hydrodynamic processes taking place in a separating element of a specified design. In this consideration, the only independent parameter will be Q_0. All other quantities, both internal (ln q, θ_V) and external (ε, L') must be determined by solving the problem simultaneously as a function of Q_0.

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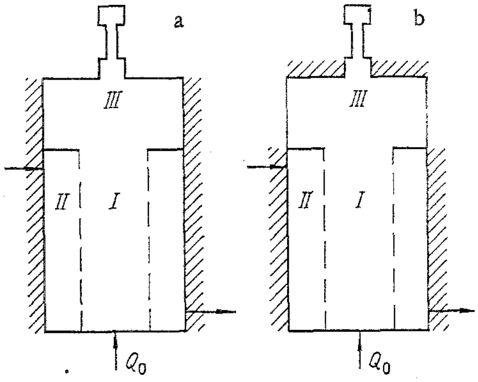

Fig. 1. Scheme of a separating element with different variants of the arrangement of the main and auxiliary coolers (the condensation surfaces are indicated by shading).

In the design, we can distinguish two fundamental variants of a separating element, which differ in the relative positions of the main and auxiliary coolers (Fig. 1).

We consider, first of all, the processes taking place in an element with a vertically placed auxiliary condenser, for the case of a fairly large specific heat extraction from the condensing surfaces.

The presence of a stream of vapor leads to a redistribution of the gas which originally filled the entire element. Part of the gas (which is lighter than the vapor) is forced into the upper cavity III and forms a cushion delimiting the surface, while the other part is forced into the gap between the diaphragm and the main condenser and into the connecting pipes. In cavity I the concentration of the gas in the vapor is maintained only as a result of its diffusion through the diaphragm; for ordinary regimes of operation of the elements this concentration is low in comparison with the concentration of gas in the other parts of the apparatus.

The motive force of the mass-transfer process is the forced stream of vapor to the condensing surface and the appearance of a partial gas pressure drop. The gas diffusing into cavity I creates an excess pressure in it, and consequently a pressure drop at the diaphragm, the value of which depends on the parameters of the external capillary as well.

Obviously, for any combination of elements in cascade, the pressure drop at the capillary is equal to the pressure drop at the diaphragm. This fact has a considerable influence on the distribution of the vapor between the main and auxiliary condensers. The vapor flow Q_T condensing on the main condenser consists of a diffusing component, due to the drop in the partial pressure of the vapor, and a convective component, which depends on the drop in the total pressure at the diaphragm. The remaining part of the vapor, Q_L, enters cavity III and diffuses in a radial direction to the surface of the upper condenser that is free of the cushion. The quantity Q_T, which defines ln q, makes the main contribution to the separating effect in the element, but both quantities, Q_T and Q_L, determine its productivity.

The picture described above enables us to create a model of a separating element which reflects fairly completely the processes taking place in it. The diffusion in the mass-diffusion element is described by the equation

$$\tau_x = -nD_{10}\frac{d\gamma}{dx} + u\gamma. \tag{1}$$

whose solution has the form [1]:

$$\tau_x = \frac{nD_{10}}{l_e}\ln q\,\frac{\gamma_a - q\gamma_b}{q-1}, \tag{2}$$

where

$$\gamma_a = \gamma_0\frac{l_e}{a}\frac{1}{\ln q}\left(1 - \frac{1}{q^{a/l_e}}\right); \tag{3}$$

$$\gamma_b = \gamma_a \left[1 - \left(1 - \frac{u\Pi y}{Q_0}\right)^{\frac{1}{q-1}}\right]; \tag{4}$$

γ_a and γ_b are the gas concentrations in the vapor–gas mixture in cavities II and I, respectively, and $\ln q = ul_e/nD_{10}$.

Integrating expression (2) over the height of the working segment of the diaphragm, taking account of (3) and (4), we obtain an equation for the flow of the light fraction:

$$L' = \int_0^{H_{\text{low}}} \tau_x dy = \gamma_a u\Pi H_{\text{low}} \frac{\ln q}{q-1}\left\{1 - q\left[1 + \frac{Q_0}{u\Pi H_{\text{low}}} \frac{q-1}{q}\left[\left(1 - \frac{u\Pi H_{\text{low}}}{Q_0}\right)^{\frac{q}{q-1}} - 1\right]\right]\right\}. \tag{5}$$

Taking account of the obvious relation

$$u\Pi H_{\text{low}} = Q_{\text{T}} - L' \tag{6}$$

we can represent Eq. (5) in the form

$$L' = \gamma_a Q_0 \left(1 - \frac{Q_{\text{T}} - L'}{Q_0}\right)\left[1 - \left(1 - \frac{Q_{\text{T}} - L'}{Q_0}\right)^{\frac{1}{q-1}}\right]. \tag{7}$$

Equations (6) and (7) provide two relations for the unknown quantities L', Q_T, and u (the quantities q and γ_a are expressed in terms of u). By solving the hydrodynamic problem of the flow of the vapor–gas mixture through the openings in the diaphragm and the flow of the light fraction through the external capillary, we can obtain another relation between L' and u.

According to [2], the flow rate through the openings of the diaphragm has the form

$$v = \sqrt{\frac{2\Delta P_{\text{d}}}{\rho}} \Big/ \left(\frac{F_1}{\alpha F_0} - 1\right), \tag{8}$$

where $\alpha = 0.63 + 0.37(F_0/F_1)^3$ is the coefficient of restriction of the flow. Taking account of the fact that the density of the vapor–gas mixture is expressed by the relation

$$\rho = \mu_{\text{v}} n\,[1 - (1 - \mu_{\text{g}}/\mu_{\text{v}})\bar{\gamma}_{\text{d}}], \tag{9}$$

where μ_v and μ_g are the molecular weights of the vapor and the gas, respectively, the gas concentration averaged over the diffusion length is

$$\bar{\gamma}_{\text{d}} = \frac{\gamma_a - \gamma_b}{\ln q} - \frac{\gamma_a - q\gamma_b}{q-1},$$

and u = nv, we obtain

$$u = \sqrt{\frac{2n\Delta P_{\text{d}}}{\mu_{\text{v}}[1 - (1 - \mu_{\text{g}}/\mu_{\text{v}})\bar{\gamma}_{\text{d}}]}} \Big/ \left(\frac{F_1}{\alpha F_0} - 1\right). \tag{10}$$

Starting from the picture of the process described at the beginning, we can replace the pressure drop at the diaphragm, ΔP_d, with the pressure drop at the external capillary. Taking account of the fact that the molar flow of gas passing through the capillary is determined by the expression

$$L' = \frac{\pi r_{\text{c}}^4 n_{\text{c}}}{8\eta l_{\text{c}}} \Delta P_{\text{c}}, \tag{11}$$

and, replacing the ratio n/n_c, to some degree of approximation, with the ratio of the temperatures in the capillary and in the element, we can reduce Eq. (10) to the form

$$u = \sqrt{\frac{16\eta l_{\text{c}} T_{\text{c}} L'}{\pi r_{\text{c}}^4 \mu_{\text{v}} [1 - (1 - \mu_{\text{g}}/\mu_{\text{v}})\bar{\gamma}_{\text{d}}] T}} \Big/ \left(\frac{F_1}{\alpha F_0} - 1\right). \tag{12}$$

Equations (6), (7), and (12) must be supplemented by an expression for the temperature of the vapor–gas mixture, which depends on the pressure in the element. For this purpose, we shall consider the processes taking place in the auxiliary condenser, and, in particular, the behavior of the gas cushion under different operating conditions.

The amount of gas in the cushion is expressed by the relation

$$m_{\text{cu}} = \pi x_{\text{c}}^2 h P \mu_{\text{g}} / R T_{\text{cu}}. \tag{13}$$

In an analogous manner, we can write the expression for the amount of gas in the external pipes:

$$m_{\text{p}} = V_{\text{p}} P \mu_{\text{g}} / R T_{\text{c}}. \tag{14}$$

If we disregard the gas content of the other regions, the sum of m_{cu} and m_p will be the amount of gas in the system up to the start of operation at room temperature and pressure:

$$\pi x_c^2 h P\mu_g/RT_{cu} + V_p \quad P\mu_g/RT_c = V_0 P_0 \mu_g/RT_c, \tag{15}$$

and hence

$$h = \left(\frac{V_0}{\pi x_c^2}\,\frac{P_0}{P} - \frac{V_p}{\pi x_c^2}\right)\frac{T_{cu}}{T_c}. \tag{16}$$

In real apparatuses the ratio T_{cu}/T_c is close to unity.

If we denote by H_{up} the height of the auxiliary condenser, the difference $H_{up} - h$ will be the segment open for the condensation of the vapor that did not pass through the diaphragm. This vapor diffuses through a narrow layer of gas, with thickness δ, which is at the surface of the condenser, and its numerical value is expressed by the formula [3]

$$Q_0 - Q_T = \frac{nD_{10}}{\delta}\,\Pi\,(H_{up} - h)\ln\frac{\gamma_0}{\gamma_{up}}. \tag{17}$$

For δ we can take, with sufficient accuracy, the value of the gap between the diaphragm and the condenser.

Eliminating the quantity h from (16) and (17), we obtain an expression for P:

$$P = \frac{P_0 V_0}{V_p + \pi x_c^2\left[H_{up} - \frac{Q_0 - Q_T}{\Pi \ln(\gamma_0/\gamma_b)}\,\frac{\delta}{nD_{10}}\right]}. \tag{18}$$

The quantity P can be related to the temperature by using an experimental relation for the pressure of saturated vapors from a vapor generator. In particular, for mercury vapors this relation can be approximated by the formula [4]

$$\lg P = 7.20 - \frac{2760}{T}. \tag{19}$$

As a result, Eqs. (6), (7), (12), (18), and (19) constitute a complete system for determining u, Q_T, L', P, and T as functions of Q_0. Knowing these quantities, we can also determine ln q and θ_v, and therefore ε as well:

$$\varepsilon = \varepsilon_0 \frac{q \ln q}{q-1}\,\frac{1}{\theta}\ln\frac{1}{1-\theta}. \tag{20}$$

A comparison of the calculated and experimental data was carried out for elements operating with mercury as the vapor generator in the separation of isotopes of neon. The elements had a cylindrical diaphragm with a diameter of 5.8 cm, a height of 15 cm, and a diffusion resistance of 1.3 cm (50 openings per cm^2, each with a diameter of 0.014 cm). The height of the upper condenser was 15 cm, and the parameters of the capillary were $r_c = 0.14$ cm and $l_c = 5$ cm.

In Fig. 2a-c we show the experimental and calculated data for Q_T, ΔP, and ε as functions of Q_0. The value of the pressure drop at the capillary is a directly observable quantity which, according to formula (11), is proportional to the flow of the light fraction. The comparison we made indicates satisfactory agreement between the calculated and experimental data, which confirms the applicability of this model to practical calculations.

In conclusion, we shall consider an element in which the surface of the auxiliary condenser is horizontal (Fig. 1b). In this case the vapor which did not pass through the diaphragm, Q_L, diffuses in a vertical direction through the entire layer of gas forced into the upper region. The diffusion equation for this case has the form

$$\tau_y = nD_{10}\frac{d\gamma}{dy} + \frac{Q_L}{S_{up}}\gamma, \tag{21}$$

where S_{up} is the area of the upper condenser. Solving this equation with boundary conditions $\gamma(H_{low}) = \gamma_b$ and $\gamma(H_{up}) = 1$, we obtain a relation for the partial pressure of the gas in the upper condenser as a function of the height:

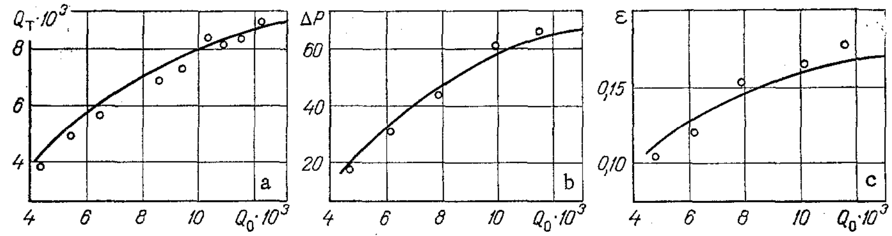

Fig. 2. Vapor flow (a), pressure drop at the capillary (b), and enrichment coefficient (c) as functions of the total vapor flow (the curves are calculated curves; Q_T, Q_0 are expressed in moles/sec, and ΔP in pascals).

$$P_g = P\left\{\frac{L'}{Q_L} + \left(\gamma_b - \frac{L'}{Q_L}\right)\exp\left[-\frac{Q_L(y - H_{low})}{nD_{10}S_{up}}\right]\right\}. \tag{22}$$

Next we determine the amount of gas in this region:

$$m_{gup} = \frac{\mu_g S_{up}}{RT}\int_{H_{low}}^{H_{up}} P_g\,dy = \frac{\mu_g P S_{up}}{RT}\left\{\frac{L'}{Q_L}H_{up} + \left(\gamma_b - \frac{L'}{Q_L}\right)\frac{nD_{10}S_{up}}{Q_L}\left[1 - \exp\left(-\frac{Q_L H_{up}}{nD_{10}S_{up}}\right)\right]\right\}. \tag{23}$$

Using the balance between the amount of gas before the start of operation and the amount under operating conditions, we obtain an expression for the pressure in the element:

$$P = \frac{P_0 V_0}{V_p\ \frac{T}{T_0} + S_{up}\left\{\frac{L'}{Q_L}H_{up} + \frac{nD_{10}S_{up}}{Q_L}\left(\gamma_b - \frac{L'}{Q_L}\right)\left[1 - \exp\left(-\frac{Q_L H_{up}}{nD_{10}S_{up}}\right)\right]\right\}}; \tag{24}$$

as in the preceding case, Eqs. (6), (7), (12), (19), and (24) form a complete system for determining all the working parameters of the separating element as functions of the flow of incoming vapor.

NOTATION

ε, enrichment coefficient; L', flow of light fraction; ln q, diffusion Peclet number; θ_v, coefficient of vapor distribution; θ, coefficient of separation of the gas flow; D_{10}, coefficient of diffusion of the gas in the vapor; Q_0, total vapor flow entering the apparatus; Q_T, vapor flow passing through the diaphragm; Q_L, vapor flow entering the auxiliary cooler; u, velocity of the vapor–gas mixture; γ, concentration of gas in the vapor; n, molecular density of the gas; τ_x, density of the gas flow; Π, perimeter of the diaphragm; H_{low}, height of the working part of the diaphragm; H_{up}, height of the auxiliary condenser; x_c, radius of the auxiliary condenser; h, height of the gas cushion; F_0, area of one opening of the diaphragm; F_1, area of the diaphragm closed to the vapor flow in the calculation for a single opening; r_c and l_c, radius and length of the capillary; ΔP, pressure drop at the capillary; P_0 and P, initial and operating pressure; T_0 and T, initial and operating temperature; V_0, volume of the element; m_{cu} and m_p, mass of gas in the gas cushion and in the pipes.

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